Robustness of semantic encoding effects in a transfer task for multiple-strategy arithmetic problems

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Abstract

The nature of the quantities involved in arithmetic problems promotes semantic encodings that affect the strategy chosen to solve them (Gamo, Sander, & Richard, 2010). Such encoding effects might prevent positive transfer to problems sharing the same formal mathematical structure (Bassok, Wu, & Olseth, 1995). In this study with 5th and 6th graders, we investigated the conditions promoting positive and negative transfer in arithmetic problems that could be solved with two distinct strategies. We showed that basic training cannot overcome the initial impact of semantic encodings, and we provided evidence that a lack of semantic encoding of the training problems leads to transfer errors. This suggests the existence of ontological restrictions on the representation mechanisms involved in problem solving tasks.

Keywords: arithmetic problem solving; analogical transfer; semantic structures; semantic alignment; strategy choice

Introduction

Semantic content in arithmetic problem solving

It is well established that the semantic content of arithmetic word problems can influence their difficulty. For example, in one-step subtraction problems, when the question bears on the final result, change problems (e.g., "John had 8 marbles, he loses 5 marbles during recess. How many marbles does John have now?") are easier to solve than combine problems (e.g., "John and Tom have 8 marbles altogether, Tom has 5 marbles. How many marbles does John have?") (Riley *et al.*, 1983). In the case of conceptual rewording, providing semantic cues relevant to the solution facilitates the construction of an appropriate mental representation and makes the problem easier to solve (Vicente, Orrantia, & Verschaffel, 2007). Success depends on how the semantic relations evoked by the entities of the problem situation are aligned with the mathematical relations of the problem (Bassok, Chase, & Martin, 1998).

Change in encoding and choice of strategy

Any problem can be described in terms of its semantic dimensions(for example, a state of a problem can be static or dynamic, discrete or continuous) ; those, in turn, influence the representation of the problem as well as the solution strategies (De Corte et al., 1985; Bassok & Olseth, 1995). Indeed, encoding can influence not only the difficulty of a problem but also the strategy employed to solve it (Brissiaud & Sander, 2010). Interestingly, some particular encodings of a problem might be more efficient than others, in terms of number of steps necessary to reach the solution. This is the case for distributive word problems (Coquin-Viennot & Moreau, 2003) or multiple-step arithmetic word problems (Thevenot & Oakhill, 2005). For example, Coquin-Viennot and Moreau (2003), gave elementary school children (grades 3 and 5) problems that could be solved either by a distributed strategy (e.g., $k \times a + k \times b$), or by a factorized strategy (e.g., $k \times (a + b)$); the presence of a word cueing for element grouping increased the frequency of the factorized strategy.

Gamo, Sander & Richard (2010) showed that the type of quantities used in arithmetic problems can determine which of the following two relationships will be emphasized: (1) the *complementation relation,* priming the computation of the difference between a whole and one of its component parts, or (2) a *matching relation,* leading to the computation of the difference between homologous quantities. Consider, in this respect, the following two problems: (a) "In the Richard family, there are 5 persons. When the Richards go on vacation with the Roberts, they are 9 at the hotel. In the Dumas family, there are 3 fewer persons than in the Richard family. The Roberts go on vacation with the Dumas. How many will they be at the hotel?" and (b) "Antoine took painting courses at the art school for 8 years and stopped when he was 17 years old. Jean began at the same age as Antoine and took the course for two years less. At what age did Jean stop?" Both can be solved by the same two strategies. However, most participants would solve (a) with a complementation strategy (9 - 5 = 4; 5 - 3 = 2; 4 + 2 = 6) and almost never use the matching strategy (9 - 3 = 6) whereas for (b) the majority of participant tend to use the matching strategy (17-2=15) rather than the complementation strategy (17-8= 9; 8-2= 6; 9+6=15) (Gamo *et al.,* 2010).

An important difference in the semantic content of the problems that could potentially account for this influence in strategy, is that (a) promotes *cardinal* (absolute) encoding, which would imply that to reach the total number of persons, the number of people in each of the component families should be known, and so these quantities are calculated first. By contrast, (b) promotes *ordinal* (relative) encoding, which implies equivalence of course duration difference and age difference. Therefore the matching strategy is already implied in the encoding step in (b); this is not the case in (a), where using the matching strategy would require an extra recoding step (Gamo *et al.*, 2010).

Overall, (a) and (b) can be said to parallel two kinds of semantic alignment as the semantic relations evoked by the entities of the problem situation are aligned with two different kinds of mathematical relations (complementation or matching relations). Cardinal encoding emphasizes the complementation relations while ordinal encoding emphasizes the matching relations, and these are associated with different solving strategies: complementation strategy or matching strategy.

The semantic determinants of transfer

Transfer from source problems to target problems has been shown to be more effective when surface features –those that can be manipulated without modifying the solution or the solving procedures, remain unchanged (e.g. Novick & Holyoak, 1991). Bassok and Olseth (1995) showed that surface features not only influence structural ones, but may also induce a semantic structure that could be congruent or incongruent with the mathematical one. Surface features appear to be instantiations of abstract semantic dimensions such as symmetry-asymmetry. Analogical transfer was shown to be influenced by these dimensions. Permutation problems with symmetric sets of elements (for example, *doctors from Chicago* and *doctors from Minnesota* are symmetric because they have equivalent semantic roles in the world) were not considered to be of the same type as permutation problems with asymmetric sets (for example, *prizes* and *students* are asymmetric, because prizes may be given to students but not vice versa). As a consequence, performance on the test problems is influenced by the specific surface features encountered in the training set (Bassok, Wu, & Olseth, 1995).

Goal of the present study

Most of the studies on transfer use problems in which only one strategy is successful. Unfortunately, failure to transfer is expressed only as failure to solve the problem, and there is no way to dissociate between the two, which may have different causes. Failure to solve the problem might result from a poor representation of the problem or from failure to match the source and the target appropriately despite the existence of an adequate representation of the problem. In order to allow us to distinguish between representational aspects and strategic ones, in the current study, we used arithmetic problems that could be correctly solved with the two distinct strategies presented above: the complementation strategy (3 steps) or the matching strategy (1 step). This allows us to dissociate positive transfer of the taught strategy from a successful resolution based on the other available strategy which also leads to a correct solution. The latter relies on another representation of the problem than the one that would lead to transfer of the strategy.

In the present study, participants knew the elementary arithmetic operations and their mathematical meaning (i.e., they knew how to add or to subtract, and what it meant to look for the value of a part or a whole, or to compare quantities). The main goal was to study their ability to transfer a new solving strategy in various contexts.

In contrast, most studies in the literature use quite complex problems (for example, permutation problems (Ross, 1989; Bassok & Olseth, 1995)). This renders the origin of transfer failures unclear. Did participants understand the meaning of the algorithms they were provided with? Is it possible that they "blindly" applied the algorithms from the source problem with very poor understanding of the underlying mathematical features? If they failed to understand the meaning of the algorithms, could they have mapped the training problem on the transfer items on the basis of perceived equivalence of roles (the reasoning 'This entity in the training problem has the same role as that entity in a transfer problem, so I should give them the same role in the algorithm'). Such questions should be answered to exclude inappropriate encoding of the training situation as the main source of failure.

We hypothesized that the transfer of the matching strategy to novel problems sharing the same formal mathematical structure should be influenced by the type of representation induced by the problems. We trained pupils on examples of the matching (1 step) strategy, and then asked them to use it in several types of problems, which varied with respect to their similarity to the example problems.

We designed our experiment to study the transfer of the matching strategy to ordinal problems, where it is spontaneous, and to cardinal problems, where the complementation strategy is spontaneous. We chose to teach the matching strategy rather than the complementation strategy, because it is more efficient as it involves a single step.

Presentation of the problems

All of the problems had the same formal mathematical structure as the ones used in Gamo *et al.* (2010), presented in Figure 1.

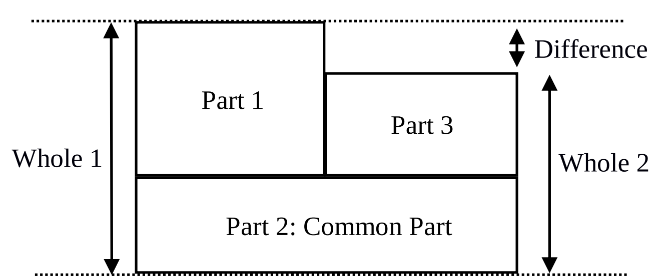

Figure 1: Formal mathematical structure of the problems.

Table 1: Presentation of the versions of the problems.

	Cardinal quantities	Ordinal quantities
Common to all problems	A bag of potatoes weighs 5 kilograms. It is weighed with a pumpkin. The weighing scale indicates a total of 11 kilograms. The same pumpkin is weighed with a bag of carrots.	Sophie's travel lasts for 5 hours. Her trip happens during the day. When she arrives, the clock indicates it's 11a.m. Fred leaves at the same time as Sophie did.
V0: identical to the source	The weighing scale indicates 2 kilograms less than before. How much does the bag of carrots weigh?	He arrives 3 hours earlier than she does. How long does Fred's travel last?
V1: inverted operands (question bearing on a whole instead of a part)	The bag of carrots weighs 2 kilograms less than the bag of potatoes. What is the weight indicated by the weighing scale?	His is 3 hours shorter than Sophie's. At what time does Fred arrive?
V2: inverted operator (addition instead of subtraction)	The weighing scale previously indicated 2 kilograms less than it does now. How much does the bag of carrots weigh?	Sophie arrives 3 hours earlier than Fred does. How long does Fred's travel last?
V3: inverted operator and inverted operands	The bag of potatoes weighs 2 kilograms less than the bag of carrots. What is the weight indicated by the weighing scale?	Sophie's travel is 3 hours shorter than Fred's. At what time does Fred arrive?

In the previously mentioned examples (a) and (b), Part 1 corresponded to the Richards *(a)* and the duration of Antoine's course *(b)*, Part 2 corresponded to the Roberts *(a)* and the age of the two children starting the course *(b)*, Part 3 corresponded to the Dumas *(a)* and the duration of Jean's course *(b)*, Whole 1 corresponded to the Richards and the Roberts *(a)* and the age of Antoine after the course *(b)*, and Whole 2 corresponded to the Roberts and the Dumas *(a)* and the age of the Jean after the course *(b)*, .

We introduced variations between problems to slightly modify the solving algorithm without changing the mathematical structure of the problems (see Table 1).

Hypotheses

Firstly, we hypothesized robustness of encoding effects: it should be more difficult to transfer the matching strategy to cardinal problems than to ordinal problems. With respect to the variations of the required algorithm (Table 1), our hypotheses were:

- (H1): Even if a literal application of the example algorithm leads to success, robust encoding effects should be observed and thus the matching strategy should be transferred less often when the quantities promote cardinal encoding than when they promote ordinal encoding.
- (H2): When the problem test varies with respect to the target of the question (H2a), or the sign of the difference (H2b) or both (H2c), participants should show more aptitude to use the matching strategy in the case of congruent (ordinal) encoding than incongruent one (cardinal encoding).

Secondly, we investigated the possible causes of negative transfer. We hypothesized that failure to solve the modified problems could mainly be explained by poor semantic encoding of the examples, manifested in a non-semantic use of the taught algorithm; namely, a literal transposition of this algorithm. We thus expected that when the test problems differed from the training problems regarding the target of the question (H3a), the sign of the difference (H3b) or both (H3c), we would observe some errors of participants failing to adjust the algorithm accordingly, indicating that these participants did not properly encode the situation, and were not able to extract the conceptual structures from the training problems.

Methods

Participants

Participants were 110 children (M=11.1 years, SD=7.8 months, from 9.5 to 13.3 years, 5th and 6th grades) attending school in the Paris area. They were recruited from 7 different classes in 6 different schools, and came from various socioeconomic backgrounds. They participated voluntarily and were not aware of the hypotheses being tested.

Design

Each child was presented with a set of problems consisting of 2 training problems and 8 test problems. All of the training problems involved an *ordinal* quantity; they were duration problems emphasizing the ordinal coding as described by Gamo et al. (2010). Three bimodal factors varied across problems: First, the **nature of the quantity** (cardinal versus ordinal). There were 4 types of quantity: two cardinal (price and weight); and two ordinal (distance and temperature) quantities. Second, the **target of the question** (part versus whole): there were four problems in which the difference between the two wholes was provided and participants had to find the unknown part; in the other four problems the difference between the two parts was given, and subjects had to find the unknown whole. Third, the **sign of the difference** (+/-): the second of the two elements could either be larger or smaller than the first one, this requiring subjects to perform a subtraction or an addition when using a matching strategy.

Materials

The problems were printed in booklets. The front page displayed the two training problems and provided the matching strategy solution for each of them. The following instructions were given on the upper-side of the page: “You will find an arithmetic problem on every page of this booklet. We ask you to take the time to thoroughly read the problems: there is no time constraint. Please write down every operation you do in order to reach a solution. Just below, you will find two training problems, followed by their respective solutions. Every other problem in this booklet can be solved using the same principle, with only one operation.”

Each test problem page was divided in three parts: the problem itself was presented on the upper-left side of the page, the response area was on the upper-right side of the page, and an area that could be used as a draft was on the bottom of every test page. These test pages were always presented on the right side of the booklets, while the two training problems with their solution with the matching strategy were displayed on each left page, in sight during the test phase as a reminder.

Procedure

The children were given the booklets and asked to read carefully the front page before starting to solve the problems. After they had answered each of the 8 problems[1], their booklets were collected. They were told to take all the time they needed; no participant exceeded 1 hour.

Coding and scoring

A problem was considered as correctly solved when the exact result was found and accompanied by the appropriate calculations. The successful strategies were categorized (correct matching, correct complementation) and so were the incorrect ones (matching with inverted operator, matching with inverted operands, matching with inverted operator and inverted operands, complementation with error, irrelevant, skipped).

For the successes, we used a success score designed to measure the distribution of matching strategies among the correct strategies: each successfully solved was given a score of 1 if solved using the matching strategy and 0 otherwise.

For the errors, we designed 3 error scores: a 'matching with inverted operator' score, a 'matching with inverted operands' score, and a 'matching with inverted operator and inverted operands' score. For each of these scores, we attributed 1 for every congruent error and 0 otherwise.

Results

Conditions of positive transfer

We first analyzed, for each problem, the proportion of matching strategies among all the correct trials (see fig. 2).

[1] Due to a reprography issue, some booklets contained only 7 problems, and thus the number of degrees of freedom isn’t always the same between our different analyses.

In order to test our first hypothesis (H1), we examined the frequency of use of the matching strategy on test problems identical to the training problems with respect to their mathematical form (same operator, same operands). Consistent with H1, participants successfully applied the matching strategy in problems eliciting an ordinal representation (success score M=0.893, SD=0.793) more often than in problems eliciting a cardinal representation (success score M=0.500, SD=0.805); this difference was significant (t(88)=3.667, p<0.001, paired t-test).

Figure 2: Proportions of correct solutions by matching strategy and complementation strategy, as a function of the similarity between the training problems and the test-problems; p-values refer to comparisons between cardinal and ordinal problems in terms of the proportion of correct matching strategies.

Similarly, we studied the results obtained for problems using the same operands as the training problems, but requiring a different operator (the sign of the difference having been changed). Consistent with H2b, success scores for cardinal trials (M=0.389, SD=0.905) were significantly lower (t(88)=2.673, p<0.01, paired t-test) than those for ordinal trials (M=0.852, SD=1.451).

Finally, we studied the use of the matching strategy when both the operands and the operator of the problems differed from those of the training problems. Again we found a significant difference (t(66)=2.453, p<0.05, paired t-test) between the success scores of ordinal problems (M=0.625, SD=1.503) and those of cardinal problems (M=0.118, SD=0.676). Problems inducing an ordinal representation therefore seemed to facilitate the use of the strategy learnt, even when it required adapting two different factors in order to be used, in conformity with H2c.

Overall, semantic encoding of the problems had a strong impact on transfer. Indeed, it is so robust that even with the example problem repeatedly shown to the participants with a solving strategy in one operation that leads to the solution, and even when the instructions explicitly state that the same solution in one operation applied to all the problems, participants tended to use the longer three-step strategy when the quantities involved promoted a cardinal encoding. In contrast, more of them used the one-step strategy when the quantities involved promoted an ordinal encoding. This holds true both when a literal application of the taught algorithm is sufficient (H1) and when this taught algorithm has to be adapted (H2).

Analysis of negative transfer

The second part of our analysis involved the distribution of errors across the experimental conditions. We created the following typology for the strategies used by participants:

- (i) correct operator with the wrong operands (calculating the whole when the question is about the part, or vice versa), classified as "inverted operands only";

- (ii) correct operands with the wrong operator (addition instead of subtraction or vice versa), classified as "inverted operator only";

- (iii) wrong operator and wrong operands, classified as "inverted operator and inverted operands".

- (iv) any other errors (use of multiplication or division, use of more than one operation leading to an incorrect result, absence of use of the difference value, use of a complementation strategy leading to a failure), classified as "other errors".

Our hypotheses did not predict a difference in the specific type of errors occurring in ordinal and cardinal problems. Indeed, there was no difference between these two groups. In the following analyses, problems were only divided according to problem type (V0, V1, V2 and V3) rather than cardinal and ordinal quantities.

We first analyzed how "inverted operands only" errors were distributed across the different types of problems (Figure 3, left). We compared test problems which were identical to the training problems (same sign of the operator and same operands) with problems in which only the choice of the operands differed from the training problem; the error scores for problems with inverted operands (M=0.629, SD=0.959) was significantly higher (t(45)=2.669, p<0.05, paired t-test) than the error scores for problems identical to the training problems (M=0.229, SD=0.605), consistent with H3a.

Regarding 'inverted operator' errors (Figure 3, middle), we compared the test trials which were identical to the training examples with the test problems which differed in terms of the operator (i.e., requiring addition rather than subtraction). Problems with an inverted operator (M=0.777, SD=1.174) had a significantly higher 'inverted operator only' error' rate (t(45)=3.439, p<0.01, paired t-test) than problems with no such change from the training ones (M=0.112, SD=0.540), consistent with H3b.

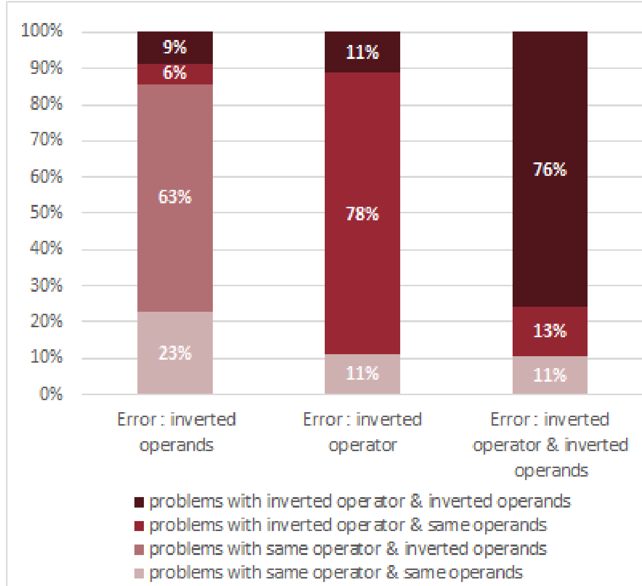

Figure 3: Distribution of the different type of errors across the problems.

Finally, we compared the proportion of 'inverted operands and inverted operator' errors (Figure 3, right) in problems homologous to the training problem and in problems with inverted operands and inverted operator. The no-change condition showed significantly less errors of this category (M=0.107, SD=0.724) than for problems with both an inverted operator and inverted operands (M=0.760, SD=1.274), (t(45)=2.911, p<0.001, paired t-test), supporting H3c.

Overall, these results suggest that participants who failed to encode the problems in an appropriate manner (either through a cardinal or an ordinal encoding) and failed to solve the problem were influenced by the algorithm shown, but applied it in a literal way. Indeed, these errors appear to have resulted from a literal transposition of the calculations provided in the example problem.

Discussion

In agreement with our hypotheses, when the quantities in the problems promoted cardinal encoding, a large proportion of participants failed to apply the algorithm they were taught to novel examples. This was true when they were sharing all the characteristics of the training problems and also when they differed in terms of the operands and/or the operator. This suggests that the representations induced by "what we know about the world" were not abolished by the explicit teaching of the matching strategy and the explicit instruction to use it. The fact that this effect persisted even when the use of the matching strategy was made less obvious by the modifications introduced between the training and the test highlights the importance of this effect.

Recent work (e.g. DeWolf, Bassok, & Holyoak, in press; Rapp, Bassok, DeWolf, & Holyoak, in press) emphasizes the generality of the phenomenon of semantic alignment and the underlying educational perspectives.

In this work we have expanded the findings of Gamo et al. (2010) that the initial spontaneous encoding constrains the spontaneous strategy. We have shown that encoding influences transfer even in situations in which the solution requires low technical knowledge (additions and subtractions) and relies conceptually on simple mathematical relations (comparison or looking for a part or a whole).

This phenomenon highlights the importance of overcoming the initial encoding in some cases, even when this initial encoding is relevant from a mathematical point of view: cardinal encoding and the associated complementation strategy were relevant for solving the problems in this study as they allowed participants to reach for the right solution. However, these have to be overcome in order to successfully apply the matching strategy. A general encoding such as the one symbolized in Figure 1 is far from spontaneous. This is a promising and challenging route towards the development of more general methods for semantic recoding which would remain compatible with the initial encoding but embrace a larger number of situations and be more mathematically apt.

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